

Mathematics Tutorial Series

Integral Calculus #10

Examples of Substitution

Example 1:

$$\int \frac{\log x}{x} dx$$

Let $u = \log x$ so that $du = \frac{1}{x} dx$.

The integral simplifies to:

$$\int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\log x)^2 + C$$

Example 2:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

Let $u = \sqrt{x}$ so that $du = \frac{1}{2\sqrt{x}} dx$ and $2 du = \frac{1}{\sqrt{x}} dx$.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$$

Example 3:

$$\int_0^1 (x^3 + 5)^6 x^2 dx$$

Let $u = x^3 + 5$ so that $du = 3x^2 dx$.

We then have $x^2 dx = \frac{1}{3} du$.

The integral simplifies to

$$\int_{\substack{x=0 \\ u=5}}^{\substack{x=1 \\ u=6}} u^6 \left(\frac{1}{3}\right) du$$

$$\frac{1}{3} \int_{\substack{x=0 \\ u=5}}^{\substack{x=1 \\ u=6}} u^6 du$$

$$\left[\frac{1}{3} \frac{1}{7} u^7\right]_{u=5}^{u=6}$$

$$\frac{1}{21} 6^7 - \frac{1}{21} 5^7$$